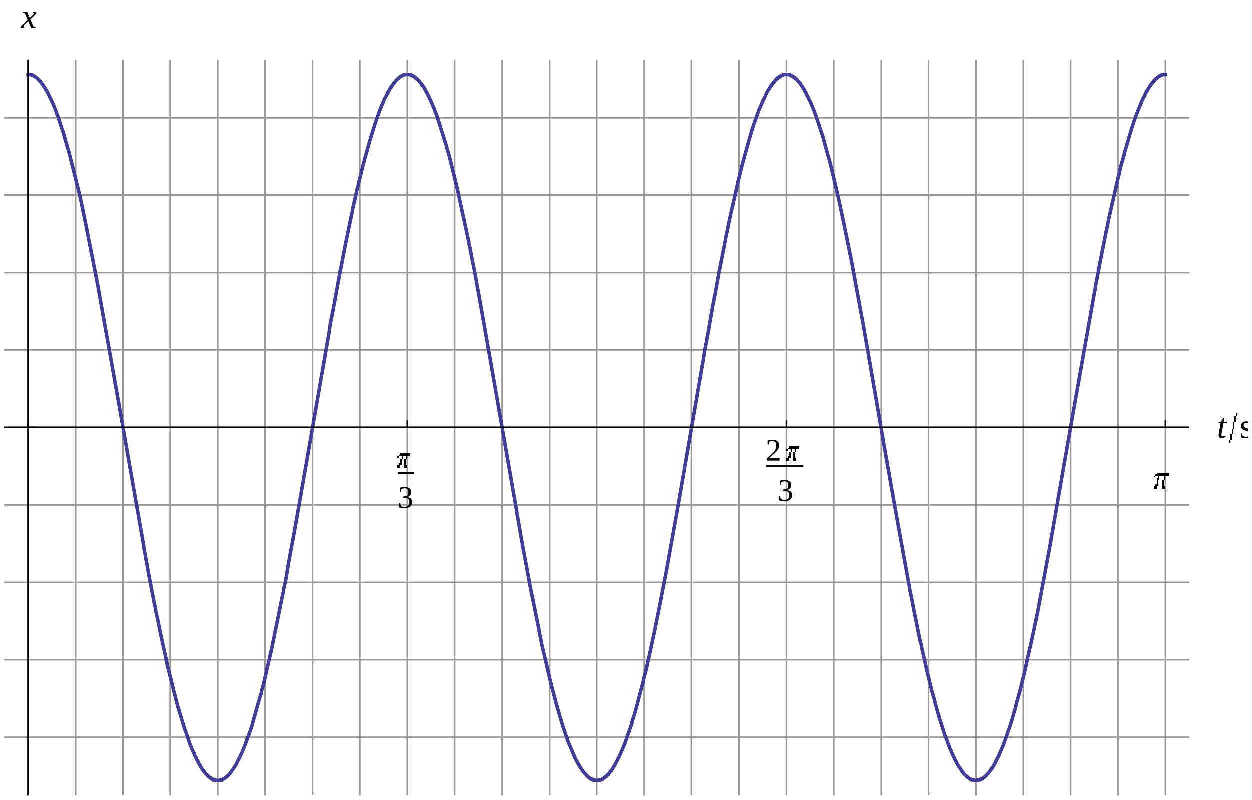
Teacher Notes

Topic C

A problem on driven oscillations

A mass of 2.0 kg is attached to a spring of spring constant 162 N m^{-1}. The system is lightly damped and is subject to a driving force. The diagram shows the variation with time of the displacement of the mass.



(a) Determine
 (i) the natural angular frequency of the system,
 (ii) the angular frequency of the driving force.

(b) The driving angular frequency is halved. On the same axes above, draw the displacement of the mass now.

Answers

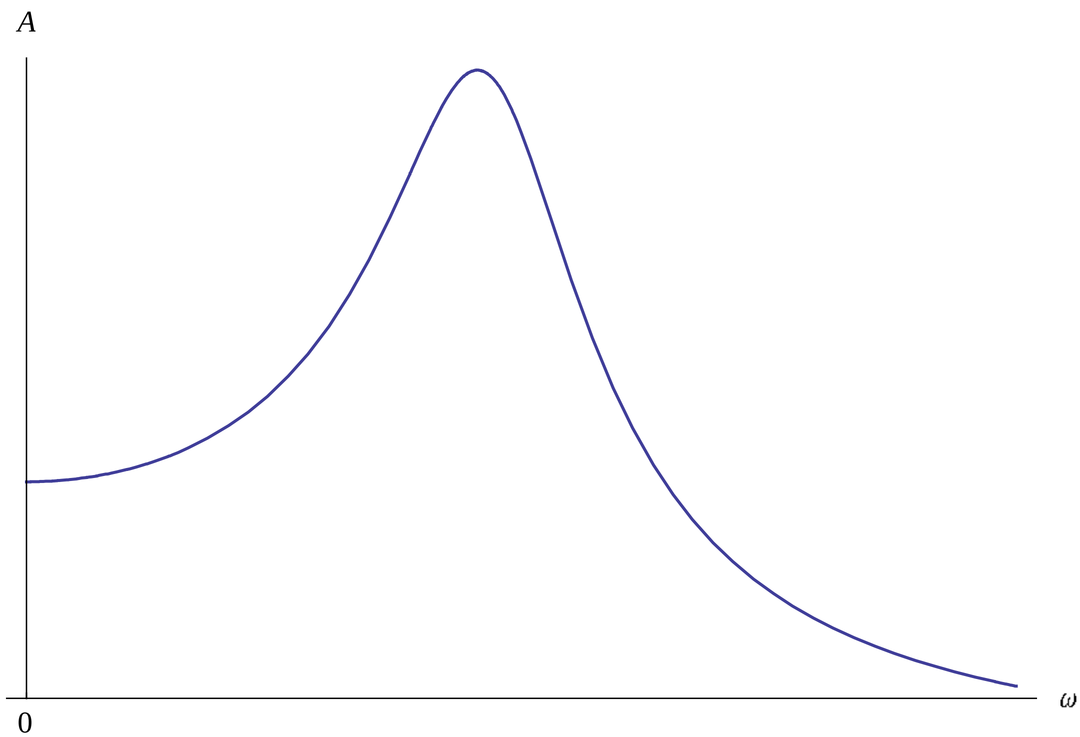
(a)

(i) The natural angular frequency is $\omega_0 = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{162}{2.0}} = \sqrt{81} = 9.0\ \text{rad s}^{-1}$.

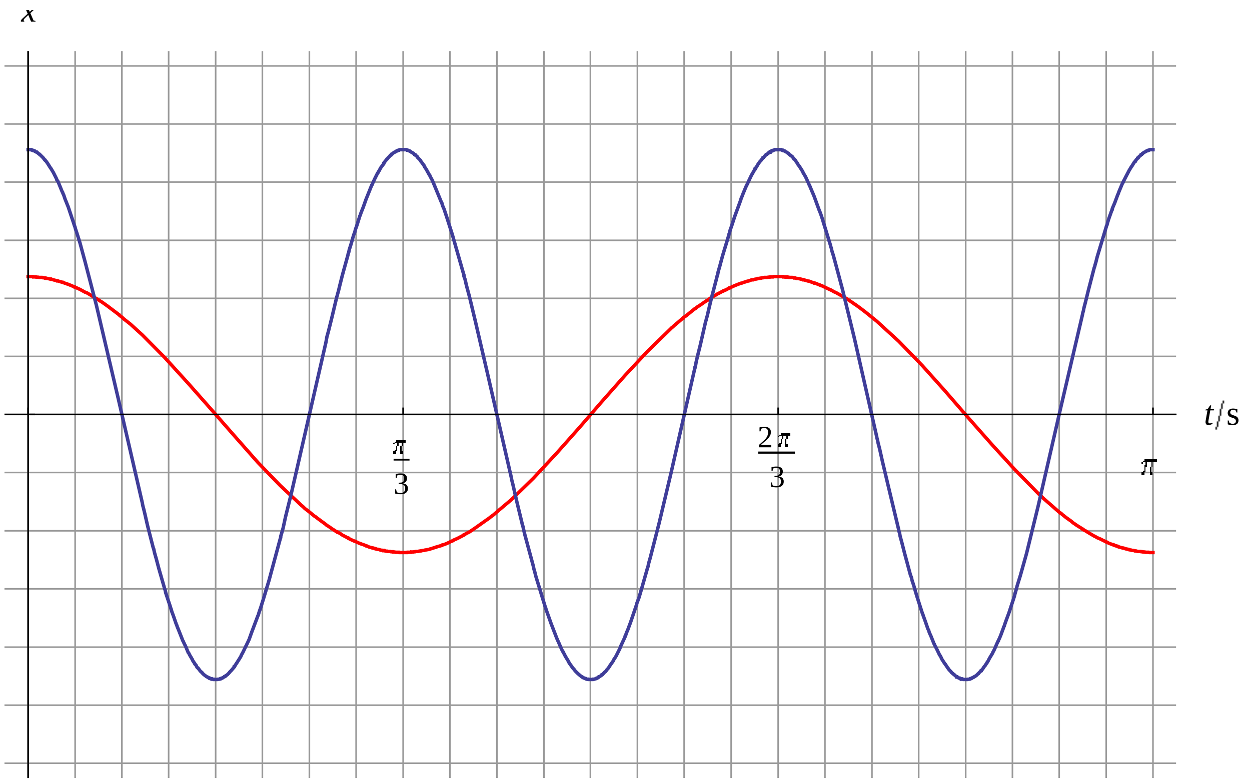
(ii) From the graph the period of the oscillations is $T = \dfrac{\pi}{3}\ \text{s}$ and so

$\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{\frac{\pi}{3}} = 6.0\ \text{rad s}^{-1}$.

(b) The new driving frequency is now $3.0\ \text{rad s}^{-1}$ and so the new period of oscillations will be $T = \dfrac{2\pi}{3}\ \text{s}$. The amplitude will be less than the original amplitude. This is because the amplitude *A* depends on the driving frequency *ω* according to the graph below.



Since the damping is low, the peak of the curve is very close to the natural frequency of 9.0 rad s^{-1}. The original frequency was 6.0 rad s^{-1} and now it is 3.0 rad s^{-1}. So, we have moved further to the left and hence the amplitude is smaller. This means we get the curve in red for the new displacement:


x
t/s
π/3
2π/3
π